INVESTIGATION 5:10

CALCULATOR KEYING

We know that multiples can be found on the calculator by using multiplication by a constant. To do this we key $\boxed{\times}$ twice in succession i.e. part of our keying sequence is $\boxed{\times}$ $\boxed{\times}$.

Multiples can also be found using a keying sequence which has the $\boxed{+}$ keyed twice in succession i.e. $\boxed{+}$ $\boxed{+}$.

Investigate.

FACTORS

A **factor** of a given number is a whole number that divides exactly into the given number.

Worked Example   Find all the factors of 12.

Answer   Since 12 is divisible by 1, 2, 3, 4, 6 and 12 then the factors of 12 are 1, 2, 3, 4, 6 and 12.

A **prime factor** is a factor that is a prime number.

Example   The factors of 12 are 1, 2, 3, 4, 6, 12.
Of these factors, only 2 and 3 are prime numbers.
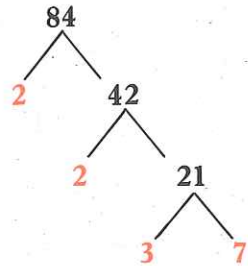The prime factors of 12 are 2, 3.

DISCUSSION EXERCISE 5:11

● Tina drew the following diagrams to find the factors of 12. **Discuss** Tina's method.

● What diagrams could you draw to find the factors of 20? **Discuss**.
   What if you were asked to find the factors of 10?
   What if you were asked to find the factors of 15?
   What if ...

● Brett used this "factor tree" to find the prime factors of 84. Brett claimed that the prime factors were 2, 3 and 7. Was Brett correct? **Discuss** Brett's method.
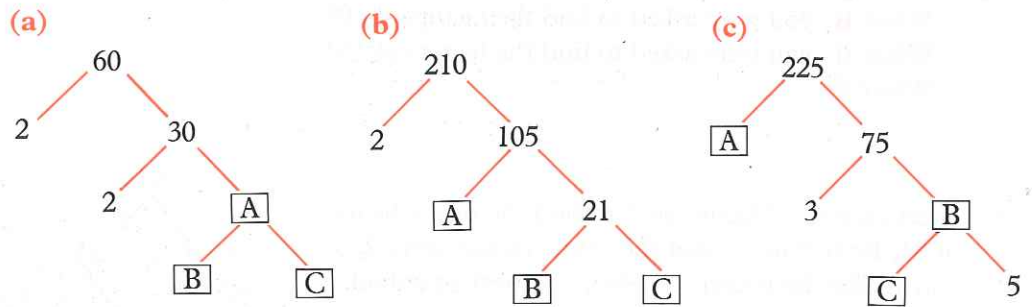

EXERCISE 5:12

1. Give all the factors of these numbers.

   (a) 6   **(b)** 8   **(c)** 10   **(d)** 13   **(e)** 15   **(f)** 28

   (g) 30   **(h)** 4   **(i)** 14   **(j)** 24

2. These lists give the factors of 6 and the factors of 15.

Factors of 6: 1, 2, 3, 6	**Factors of 15: 1, 3, 5, 15**

   (a) What two factors are common to both 6 and 15?

   (b) What is the largest common factor of 6 and 15?

3. **(a)** List the factors of 12.

   (b) List the factors of 28.

   (c) What factors are common to both 12 and 28?

   (d) What is the largest common factor of both 12 and 28?

4. Find the common factors of

   (a) 18 and 24   **(b)** 16 and 36   **(c)** 40 and 25

   (d) 6 and 18   **(e)** 20 and 36   **(f)** 32 and 80

   (g) 18 and 63   **(h)** 11 and 17.

5. Find the missing numbers A, B, C on these factor trees.

(a)

```
        60
       /  \
      2    30
          /  \
         2   [A]
            /  \
          [B]  [C]
```

(b)

```
          210
         /   \
        2    105
            /   \
          [A]    21
              /    \
            [B]    [C]
```

(c)

```
          225
         /   \
       [A]    75
      /  \    /  \
     3       [C]  5
```

```
          225
         /   \
       [A]    75
       /     /  \
      3    [C]   5
          / \
        [B]
```

6. Write down all the prime factors (i.e. the factors that are prime numbers) of these.

(a) 2   **(b)** 6   **(c)** 7   **(d)** 12   **(e)** 16   **(f)** 19

(g) 25   **(h)** 36   **(i)** 8   **(j)** 18

Start

7.

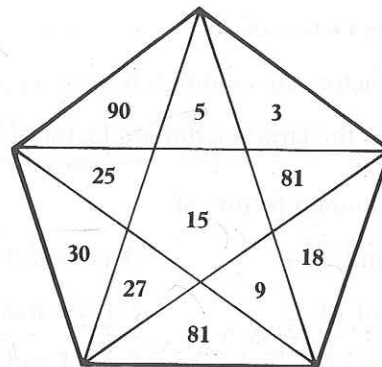| 25	50	33	10	4	6	27
21	28	15	35	63	14	15
6	39	26	12	35	9	40
45	4	3	15	42	55	35
9	8	33	38	11	77	14
12	36	35	66	99	51	75

Finish

Starting at 25 make your way down to 75. Move one square at a time through numbers which have a common factor greater than 1. You may move sideways or up or down but not diagonally. Write down the numbers you pass through.

8. Copy this diagram.
Shade all the sections that contain

(a) factors of 15

(b) square numbers

(c) multiples of 9.

Which section is not shaded?

90   5   3
25   81
    15
30   18
  27   9
    81

70

Review 1   **(a)** Write down all the factors of 48.

               (b) Write down the prime factors of 140.

               (c) Find the common factors of 14 and 35.

               (d) What is the largest common factor of 16 and 40?

Review 2

A bagpipe has the same number of pipes as the number of factors of 16.
How many pipes are in a bagpipe?

Review 3

2	3	4	5	6	7	9	12	15	21	25	28	35	36

Copy this list of numbers.   Cross out all the square numbers.

Cross out the multiples of 7.   Cross out the factors of 30.

<div align="center">Which number is left?</div>

PUZZLES 5:13

1. I am a prime number.
   I am a factor of 21.
   I am not a factor of 12.
   What number am I?

2. I am a factor of 30 and also of 48.
   I am not a factor of 15 or of 16.
   What number am I?

3. A 30 year old mother has two children at school. The age of each child is a factor of the mother's age. The sum of their ages is also a factor of the mother's age. How old are the children?

4. What is the smallest number that has exactly 5 factors?

5. What number under 100 has the largest number of factors? Is there more than one answer?

GAME 5:14

FACTOR FUN — a game for two players (A and B).

Copy this number square.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Toss a coin to see who goes first. If A wins the toss, A crosses out any number. This number is A's score. B then crosses out as many factors of this number as he or she can find and adds them. This is B's score. B then crosses out a number that is still left and adds this to his or her score.

A then crosses out as many factors of this number as he or she can find and adds these to his or her score. Then A crosses out a number that is still left and adds this to his or her score.

The game continues in this way until all the numbers have been crossed out. The winner is the one with the highest score.

For instance, if A crosses out 28 the game starts as shown.

A crosses out 28.
A's score = 28.

B crosses out 1, 2, 4, 7, 14.
B's score = 1 + 2 + 4 + 7 + 14
= 28

B crosses out 32.
B's score = 28 + 32
= 60

A crosses out 8, 16.
A's score = 28 + 8 + 16
= 52

A crosses out 17.
A's score = 52 + 17
= 69

B cannot cross out any factors of 17.
B crosses out 35.
B's score is now 95.
It is now A's turn.

JUST FOR FUN

1. **Step 1**   Write down any three-digit number.

   Step 2   Write this number down again to form a six-digit number.

   Step 3   Divide by 77.

   Step 4   Divide by 13.

             What do you notice about your answer?

2. **Step 1**   Write down a two-digit number.

   Step 2   Multiply one of the digits by 5, add 6 to the answer and double the result.

   Step 3   Add on the other digit, then subtract 12.

             What do you notice about your answer?

3. **Step 1**   Write down any multiple of 9 that is less than 91.

   Step 2   Add a number between 1 and 9.

   Step 3   Add together the digits of the number you now have.
   If this total is a two-digit number keep on adding the digits until you get a one-digit number.

             What do you notice about your answer?